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Moment Curvature Characteristics for Structural Elements of RC Building

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Abstract: The paper highlights the investigation of nonlinear axial-force and moment-curvature relationships for rectangular reinforced concrete (RC) beam and column cross sections for the reference reinforced concrete building structure considered for Probabilistic Seismic Risk Evaluation. The highly popular model namely Mander's model is used for concrete stress-strain relationship since it is simple and effective in considering the effects of confinement. The module determines the expected behavior of a user-defined cross-section by first dividing the section into a number of parallel concrete and steel "fibers". Then, the section forces and deformations are determined from the fiber strains and stresses using fundamental principles of equilibrium, strain compatibility, and constitutive relationships assuming that plane sections remain plane.

Index Terms: Mander's Method, Moment-Curvature, Stress-Strain Relationship and Moment Curvature

1. INTRODUCTION

Since the country lie in earthquake prone area and many of the destructive earthquakes occurred in the history so far resulting in high number of casualties due to collapse of buildings and dwellings. A major challenge for the performance based seismic engineering is to develop simple yet efficiently accurate methods for analyzing designed structures and evaluating existing buildings to meet the selected performance objectives Elastic analyses are insufficient because they cannot realistically predict the force and deformation distributions after the initiation of damage in the building. Inelastic analytical

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procedures become necessary to identify the modes of failure and the potential for progressive collapse. The need to perform some form of inelastic analysis is already incorporated in many building codes. Theoretical moment-curvature analysis for reinforced concrete columns, indicating the available flexural strength and ductility, can be conducted providing the stress-strain relation for the concrete and steel are known. The moments and curvatures associated with increasing flexural deformations of the column may be computed for various column axial loads by incrementing the curvature and satisfying the requirements of strain compatibility and equilibrium of forces.

2. STRESS-STRAIN MODEL FOR CONCRETE

Mander's model is highly popular model since it is simple and effective in considering the effects of confinement. It considers increase in both the strength and ductility of RC members with confined concrete. The model is popularly used to evaluate the effective strength of the columns confined by stirrups, steel jacket and even by FRP wrapping as accomplished in Figure 1.

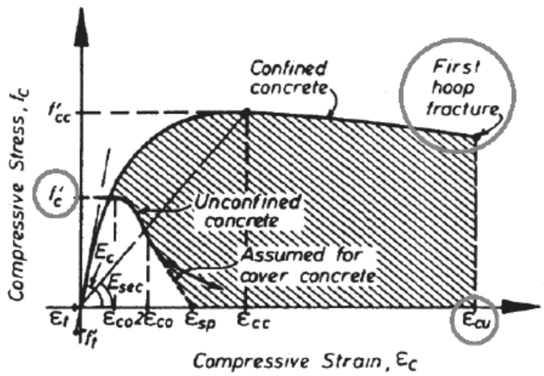

Figure 1: Mander's Model for Stress-Strain Relationship for Confined Concrete

3. STRESS-STRAIN CURVES FOR REINFORCING STEEL

The idealized stress-strain curve for concrete as recommended by IS: 456-2000 is as shown in Figure 2. Stress-strain curve for steel as per British code CP 110-1972 as shown in Figure 3, accordingly the term 0.7fy is the simplification of the expression $\left[\dfrac{fy}{\gamma_m + \dfrac{fy}{2000}}\right]$. It gives all the simplified general equations which can be used for any grade of steel.

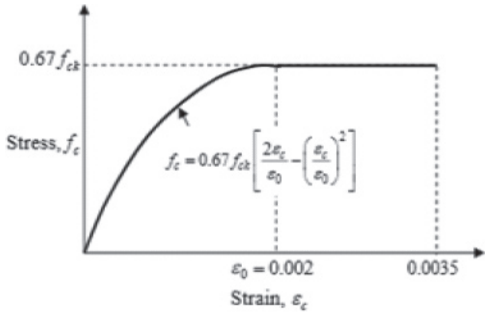

Figure 2: Stress Strain Curve for Concrete

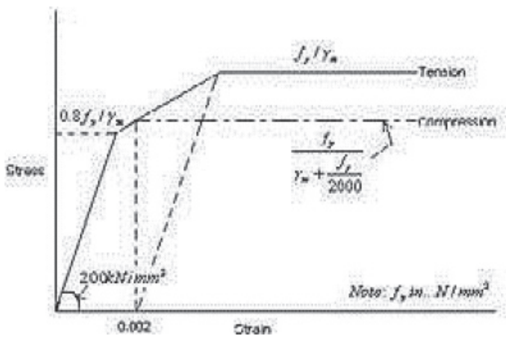

Figure 3: Stress-Strain Curve for Steel

4. STRUCTURAL SYSTEM

The building is an RC G+3 framed structure. The floor plan is same for all floors. The beam arrangement is different for the roof. It is symmetric in both the direction. The concrete slab is 120 mm thick at each floor level. Overall geometry of the structure including the beam layout of all the floors is as shown in Figure 4.

Figure 5 and 6 shows the size and reinforcement details for floor beam and roof beam sections at the column face. Figure 7 shows the size and reinforcement details for column at the beam face.

5. MOMENT CURVATURE FOR BEAM

The most fundamental requirement in predicting the Moment Curvature behaviour of a flexural member is the knowledge of the behaviour of its constituents. With the increasing use of higher-grade concretes, the ductility

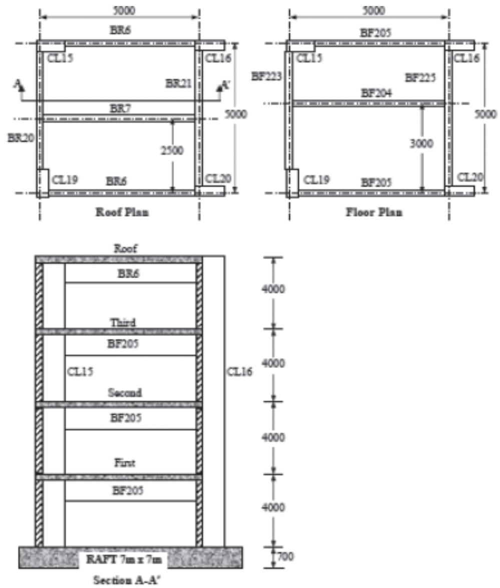

Figure 4: Overall Geometry of the Structure

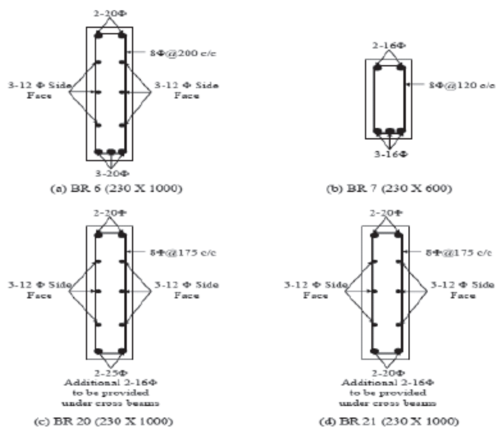

Figure 5: Details of Roof Beams

of which is significantly less than normal concrete, it is essential to confine the concrete. In a flexure member the shear reinforcement also confines the concrete in the compression zone. The relationship for the bending member as depicted in Figure 8 is as follows,

$$\frac{M}{EI} = \frac{f_c}{E_c x} = \frac{1}{R} = \varphi$$

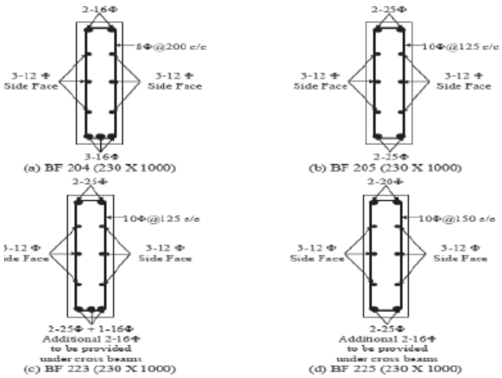

Figure 6: Details of Floor Beams

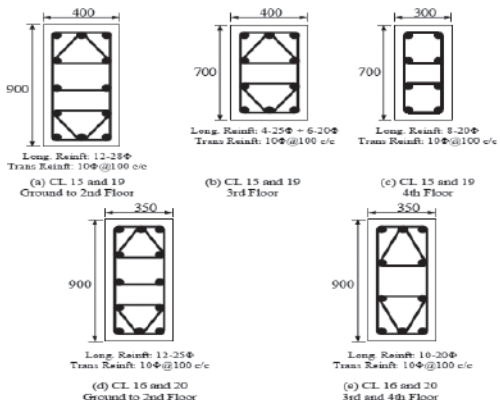

Figure 7: Details of columns at various levels

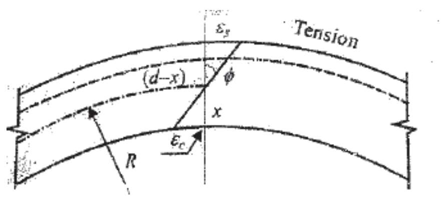

Figure 8: Beam Members in Bending

Hence,

$$\varphi = \frac{\varepsilon_c}{x} = \frac{\varepsilon_s}{d - x} = \frac{\varepsilon_c + \varepsilon_s}{d}$$

Where,

ϕ = Curvature

fc = Maximum stress in compression in concrete

ε_c = Maximum strain in concrete

ε_s = Maximum strain in steel

x = Depth of neutral axis

d = Effective depth of section

Let consider BF205 beam section as depicted in Figure 9,

$$\rho = \frac{A_s}{A_c}$$

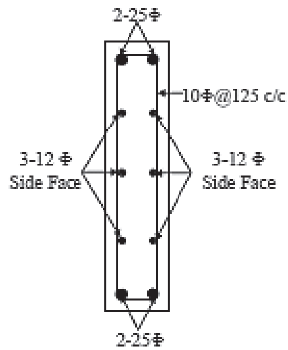

Figure 9: Detailed BF204 Beam Section

Modular Ratio (m) for elastic analysis = $\frac{280}{f_{ck}} = 14$

Curvature ϕ at cracking moment just before cracking

$$f_{cr} = 0.7\sqrt{f_{ck}} = 3.1305 \text{ MPa}$$

At just before cracking moment is resisted by concrete in tension therefore Neglecting steel, N.A. = d/2

$$M_{cr} = \frac{f_{cr} I_{gr}}{y} = 120.00 \text{ KNm}$$

$$\varphi = \frac{f_{cr}}{y} = \frac{M}{EI} = 0.00028$$

Curvature after cracking

Concrete does not take any tension so depth of neutral axis x is,

$$\mathrm{k} = \sqrt{(\rho+\rho')^2 n^2 + 2n(\rho+\rho'\frac{d'}{d})} - (\rho+\rho')n$$

$$\mathrm{k} = 0.2179$$

Effective moment of inertia

$$\mathrm{I_{eff}} = \frac{\mathrm{bx}^3}{3} + \mathrm{A_s m(D - x)^2} = 7.990\mathrm{mm}^4$$

$$\Phi \,\mathrm{at}\ \mathrm{M_{cr}} = \frac{\mathrm{M_{cr}}}{\mathrm{EI_{eff}}} = 0.00067$$

ε_s strain in steel = f_y/E_s = 0.0021

The neutral axis at yielding is given as distance kd from extreme compression fiber, where the ratio k is calculated using expression:

$$\mathrm{k} = \sqrt{(\rho+\rho')^2 n^2 + 2n(\rho+\rho'\frac{d'}{d})} - (\rho+\rho')n$$

$$\mathrm{k} = 0.2179$$

Where $\rho\left(\frac{A_s}{bd}\right)$ and $\rho'\left(=\frac{A_{s'}}{bd}\right)$ are the tension and compression steel ratios, *n* is the modular ratio, and d and d' are the distance of compression and tension steel from extreme compression fiber.

A_s Steel in tension

$A_{s'}$ Steel in compression

Depth of Neutral axis at yield stage; x = kd =208.65mm = .21m

Taking moment about compressive force due to concrete, yield moment is given by:

$$\mathrm{M_y} = A_S f_y\left(d - \frac{kd}{3}\right) + A_S' f_y'\left(\frac{kd}{3} - d'\right)$$

Since stress in the tension steel is f_y, using similar triangles, stress in compression steel is calculated as

$$\mathrm{f_y} = (\frac{d-d'}{d-kd})f_y = 99.01\mathrm{MPa}$$

Curvature is then obtained as

$$\varphi_y = \frac{\varepsilon_y}{\mathrm{d - kd}} = 0.00277$$

After yielding of tension steel, its stress remains constant but strain keeps increasing until compressive strain in extreme fiber of concrete reaches the strain value of ε_{cu} at maximum stress in concrete f_c'. In order to address the nonlinearity in concrete at high strains, Whitney-block is used to approximate the parabolic stress distribution in concrete to an equivalent rectangular stress-block representation.

The calculation of ultimate state requires iteration. For hand calculations, let us assume that strain in compression steel ε_s, exceeds the yield strain ε_y. This assumption will be checked later. Equilibrium of tension and compressive force gives the depth of neutral axis as;

$$c = \frac{\mathrm{A_s f_y} - \mathrm{A'_s f'_s}}{85\mathrm{f'}_c b\beta_1} = \frac{A_s f_y - A'_s f'_s}{36 f'_c b} = 187.3319$$

Ultimate moment is then obtained by taking moment about tension steel as:

$$\begin{aligned} M_u &= C_c\left(d - \frac{c}{2}\right) + C_S(d - d') \\ &= 36 f'_c Cb\left(d - \frac{c}{2}\right) + A'_s f'_s (d - d') \\ &= 361.57 \text{ KNm} \end{aligned}$$

$$\varphi_{\mathrm{u}} = \frac{0034}{c} = 0.01868$$

The ductility factor

$$\text{D. F.} = \frac{\varphi_u}{\varphi_y} = 6.74$$

Where is ϕ_u ultimate strain in concrete at maximum stress, which is 0.0034 as per IS456-2000. This is first trial value of ε_u. Assumption of yielding in compression is now checked by ensuring:

$$\varepsilon_s = \left(\frac{\mathrm{c - d}}{c}\right)\varepsilon_{cu} \geq \varepsilon_y$$

If the above condition is satisfied then assumption made is true and obtained value of(M_u, ϕ_u) defines the ultimate state on the moment-curvature curve. If the condition is not satisfied further iteration is required with new trial strain value as

$$\varepsilon_s = \frac{\varepsilon_y + \varepsilon_s}{2}$$

6. CONCRETE PROPERTIES

Currently, the entire cross-section is assumed to be unconfined. The compression stress-strain relationship of the unconfined concrete is determined using a method developed by Mander et al. In this method, the concrete stress, fc is given as a function of the strain, ε_c as:

$$fc = \frac{f_C{}' x.r}{r - 1 + x^r} \text{ in MPa}$$

$$x = \frac{\varepsilon_c}{\varepsilon_{co}}, \; r = \frac{E_c}{E_o - E_{sec}}$$

ε_{c0} is the strain at peak stress (f_c') and E_c is the tangent modulus of elasticity of concrete calculated as

$$Ec = 5000\sqrt{f_c{}'} \quad \text{MPa}$$

Esec, the secant modulus of elasticity, is the slope of the line connecting the origin and peak stress on the compressive stress-strain curve (i.e., $Esec = \frac{f_c{}'}{\varepsilon_{CO}}$).

Crushing of the unconfined concrete is assumed to occur at ε_{cu} $2\varepsilon_{co}$.

7. THE MOMENT-CURVATURE RELATIONSHIPS OF THE COLUMN CROSS-SECTIONS

This is an iterative process, in which the basic equilibrium requirement P = C-T is used to find the neutral axis location, c, for a particular maximum concrete compressive strain, ε_{cm} where P = axial force; C = internal compression stress resultant; and T = internal tension stress resultant. The total concrete compressive stress resultant and the location of its centroid are determined by integrating numerically under the concrete stress distribution. The bending moment is assumed to act such that the top surface of the cross-section is in compression.

The entire process can be summarized for a cross-section with two layers of reinforcing bars as depicted in Figure 10 follows

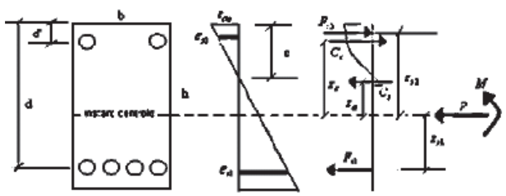

Figure 10: The Cross-Section with Two Layers of Reinforcing Bars

For example the column sections property in present work is 400mm x 900mm in size with 12-28mm dia bars placed as shown in the figure 11 below. The transverse reinforcement for the columns is provided 10mm stirrups/ties @ 100mm c/c as figure shows.

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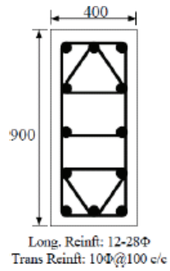


Figure 11: Detailed CL 15 and 19 Column Section (Ground to 2nd Floor)

The calculation of the following four points on a moment-curvature curve will be taken for plotting the moment curvature curve for various column section shown in this example: ε_{cm} =0.25 ε_{cu};ε_{cm} = 0.5 ε_{cu};ε_{cm} = 0.75 ε_{cu}; ε_{cm} = 1.0ε_{cu}(concrete crushing).

A. Axial Load, P

The axial load considered for sample calculations of CL15 is 50% of the balanced failure load. The balanced load, P_b, is computed as follows:

The neutral axis, c = $c_b = d \dfrac{\varepsilon_{cu}}{\varepsilon_{cu} + \varepsilon_{y}}$,

Where, $\varepsilon_y = f_y/E_s$

With εcu = 0.004 and d = 848m, c_b = 0.525m.

The concrete compressive resultant, Cc, is determined by numerically integrating under the concrete stress distribution curve.

$$\mathrm{Cc} = \int_0^{c_b} f_c b dx = \int_0^{\varepsilon_{CU}} f_c b \frac{c}{\varepsilon_{cm}} d\varepsilon_c$$

Cc = 3531.24KN

The spacing of each layer of steel is s = 0.198m

The steel forces of each layer top to bottom wise, F_{s1} and F_{s2}, F_{s3}, F_{s4} and F_{s5}, respectively, are calculated using similarity to find the strains in the layers. Balanced failure condition, by definition, has strain values $\varepsilon_{cu} = \varepsilon_{cm}$ = 0.0034 for concrete and $\varepsilon_{s5} = \varepsilon_y$= 0.002 for bottom layer of steel. For the top steel,

$$\varepsilon_{s1} = \varepsilon_{sS} \frac{c - d'}{d - c} = 0.0031$$

Similarly for second and third layer,

$$\varepsilon_{s1} = \varepsilon_{sS}\frac{c-d'-s}{d-c} = 0.0019$$

$$\varepsilon_{s3} = \varepsilon_{cm}\frac{d-c-s}{c} = 0.0008$$

$$\varepsilon_{4} = \varepsilon_{cm}\frac{d-c-2s}{c} = 0.0006$$

$$\varepsilon_{s5} = \varepsilon_{cm}\frac{d-c}{c} = 0.0021$$

This implies that the first and fifth layer of steel is at yield stress.

Hence,

$F_{s1} = f_y A_{s1} = 766.61$ KN (compression)

$F_{s2} = \varepsilon_{s2} E_s A_{s2} = 458.06$KN (compression)

$F_{s3} = \varepsilon_{s3} E_s A_{s3} = 195.47$KN (tension)

$F_{s4} = \varepsilon_{s4} E_s A_{s4} = 142.45$KN (tension) and

$F_{s5} = f_y A_{s5} = 766.61$KN (tension)

Where, A_{s1}, A_{s2}, A_{s3} A_{s4} and A_{s5} are the total reinforcing steel areas in each layer. As per IS456 consideration the concrete tensile strength in the tension region recommends the modulus of rupture to be taken as,

$$f_r' = \sqrt{f'_c} \text{ MPa for normal weight concrete.}$$

Thus, for $f'_c = 20$ MPa, $f'_r = 3.3$ MPa. The concrete tension force

$$C_t = \frac{1}{2} f'_r A_{cr}, C_t = 22.97 \text{ KN (tension)}$$

Where, A_{cr} is the area of concrete in tension calculated based on the linear strain diagram.

Then, the balanced axial load is found from equilibrium as,

$$P_b = C_c + F_{s1} + F_{s2} - F_{s3} - F_{s4} - F_{s5} - C_{cr}$$

Therefore, 50% of the balanced load used is P =1814.20kN.

B. Instant Centroid

The axial load acts at an "instant" centroid assumes location that for the calculation of the moment-curvature relationship. The location of the instant centroid is determined by assuming an initial condition where only the user-selected axial load acts on the cross-section without moment. This loading condition produces a uniform compression strain distribution throughout the cross-section.

Let the uniform compression strain be equal to ε_{ci} then,

$$P = f_{ci}A_c + f_{s1}A_{s1} + f_{s2}A_{s2} + f_{s3}A_{s3} + f_{s4}A_{s4} + f_{s5}A_{s5}$$

From equilibrium,

A trial-and-error solution is needed since it is not known in advance. Then, the location of the instant centroid, x, from the top compression face is determined as,

$$x \frac{\frac{f_{ci}A_c h}{2} + f_{s1}A_{s1}d + f_{s2}A_{s2}(d-s) + f_{ss}A_{ss}(d-2s) + f_{s4}A_{s4}(d-3s) + f_{ss}A_{ss}d'}{f_{ci}A_c + f_{s1}A_{s1} + f_{s2}A_{s2} + f_{s3}A_{s3} + f_{s4}A_{s4} + f_{s5}A_{s5}}$$

$$x = 0.4505\text{m}$$

The calculation of the first point on the moment-curvature relationship of the section can be summarized as follows:

1. $\varepsilon_{cm} = 0.25\ \varepsilon_{cu} = .00084$
2. Assume the neutral axis depth, a distance c = .4m
3. From the linear strain diagram geometry

$$\varepsilon_{s1} = \varepsilon_{s5}\frac{c-d'}{d-c} = 0.0031$$

Similarly for second and third layer,

$$\varepsilon_{s2} = \varepsilon_{s5}\frac{c-d'-s}{d-c} = 0.0019$$

$$\varepsilon_{s3} = \varepsilon_{cm}\frac{d-c-s}{c} = 0.0008$$

$$\varepsilon_{s4} = \varepsilon_{cm}\frac{d-c-2s}{c} = 0.0006$$

$$\varepsilon_{s5} = \varepsilon_{cm}\frac{d-c}{c} = 0.0021$$

4. The steel stress resultants are,

$F_{s1} = f_y A_{s1} = 282.63$ KN (comp)
$F_{s2} = \varepsilon_{s2} E_s A_{s2} = 84.79$ KN (comp)
$F_{s3} = \varepsilon_{s3} E_s A_{s3} = 129.80$ KN (tension)
$F_{s4} = \varepsilon_{s4} E_s A_{s4} = 26.17$ KN (tension)
$F_{s5} = f_y As_5 = 350.15$ KN (tension)

5. Determine Cc by integrating numerically under the concrete stress distribution curve.

$$\text{Cc} = \int_0^{c_b} f_c b dx = \int_0^{\varepsilon_{cu}} f_c b \frac{c}{\varepsilon_{cm}} d\varepsilon_c$$

Cc = 1755.09 KN

The concrete that has not cracked below the neutral axis contributes to the tension force C_t.

$$f'_r = \sqrt{f'_c} \text{ MPa}$$
$$C_t = 30.82 \text{ KN}$$

6. Check to see if

$$P_b = C_c + F_{s1} + F_{s2} + F_{s3} - F_{s4} - F_{s5} - C_t$$
$$P_b = 1585.57 \text{ KN}$$

So, the neutral axis must be adjusted downward, for the particular maximum concrete strain that was selected in Step 1, until equilibrium is satisfied. This iterative process determines the correct value of c. Trying neutral axis depth c = .431m gives,

ε_{s5} = 00084 (below yield); εs4 = 0.0006; ε_{s3} = 0.0008; ε_{s2} = 0.0019; ε_{s1} = 0.0031

And

Cc = 1917.44KN; Ct = 28.51KN;

F_{s1} =285.29KN; F_{s2} = 95.34KN;

F_{s3} =101.08KN; F_{s4} = 6.23KN;

F_{s5} = 293.91KN

Section curvature can then be found from,

$$\varphi = \frac{\varepsilon_{cm}}{c} = 0.002125$$

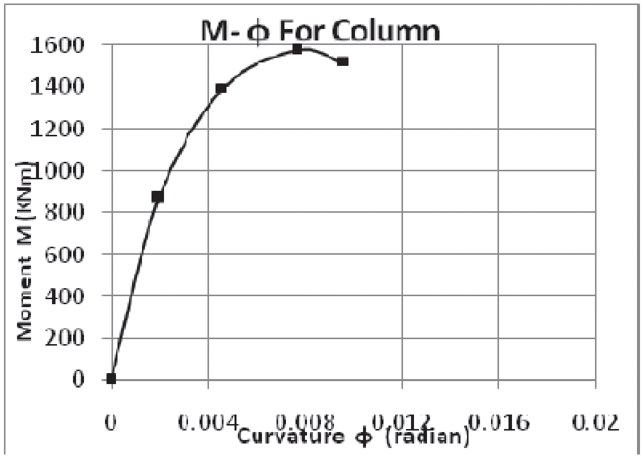
The internal lever arms for the resultant compression and tension forces of the concrete measured from the instant centroid.

Then, the section moment can be calculated as,

$$M = C_c z_c + F_{s1} z_{s1} + F_{s2} z_{s2} + F_{s3} z_{s3} + F_{s4} z_{s4} + F_{s5} z_{s5} + C_t z_t$$
$$M = 872.22 \text{ kNm}$$

Similarly curvature and moment value are also calculated at $\varepsilon_{cm} = 0.5\varepsilon_{cu}$; $\varepsilon_{cm} = 0.75\varepsilon_{cu}$; $\varepsilon_{cm} = 1.0\varepsilon_{cu}$ (concrete crushing)

The moment-curvature plot for column has been shown in Figure 12 and for beam as shown in Figure 13.



Figiure 12: Moment-Curvature Relationship Curve for Column

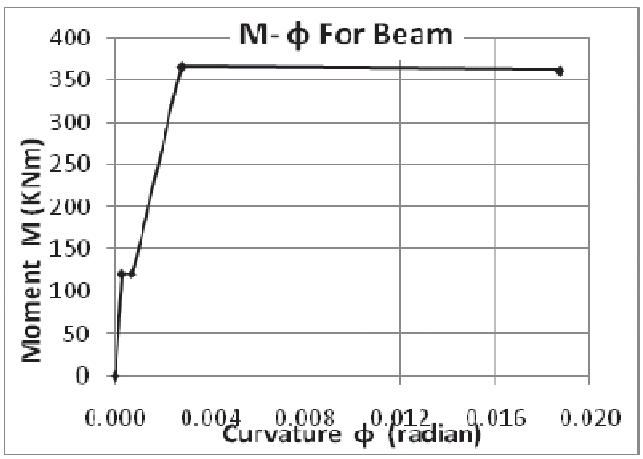


Figure 13: Moment-Curvature Relationship Curve for Beam

Table 1 shows point at moment-curvature curve for column. Table 2 shows moment at various points in beams. Table 3 shows curvature at various points in beams. Table 4 shows moment at various points in column and Table 5 shows curvature at various points in column.

Table 1. Point At Moment-Curvature Curve For Column

Point	M(moment)	Curvature ϕ
0	0	0
$0.25\varepsilon_{cu}$	872.2165	0.0019
$0.5\varepsilon_{cu}$	1395.398	0.0046
$.75\varepsilon_{cu}$	1579.791	0.0076
$1.0\varepsilon_{cu}$ (concrete crushing).	1520.344	0.0095

Table 2: Moment At Various Points In Beams

Beam	Before Cracking	After Cracking	At yield	Crushing of Concrete
BF205	120.00	120.00	365.62	361.57
BF204	120.00	120.00	153.21	154.78
BF225	120.00	120.00	237.57	238.17
BF223	120.00	120.00	366.72	363.35
BR6	120.00	120.00	237.45	238.21
BR7	43.20	43.20	88.19	88.20
BR21	120.00	120.00	236.39	236.97
BR20	120.00	120.00	237.57	238.17

Table 3: Curvature At Various Points In Beams

Beam	Before Cracking	After Cracking	At yield	Crushing of Concrete
BF205	0.00028	0.00067	0.00277	0.019
BF204	0.00028	0.00138	0.00253	0.044
BF225	0.00028	0.00096	0.00262	0.031
BF223	0.00028	0.00067	0.00275	0.020
BR6	0.00028	0.00096	0.00262	0.030
BR7	0.00047	0.00159	0.00453	0.046
BR21	0.00028	0.00096	0.00265	0.027
BR20	0.00028	0.00096	0.00262	0.031

Table 4: Moment At Various Points In Column

Column	Origin	Yield	Ultimate	Strain hardening
CL 15/19 G/2nd	0	987.64	1506.64	802.35
CL 15/19 3rd	0	440.52	673.06	228.49
CL 15/19 4th	0	272.33	406.20	217.02
CL16/20 G/2nd	0	792.80	1226.82	647.90
CL 16/20 3/4th	0	469.46	723.75	302.12

Table 5: Curvature At Various Points In Column

Column	Origin	Yield	Ultimate	Strain hardening
CL 15/19 G/2nd	0	0.0054	0.0078	0.1468
CL 15/19 3rd	0	0.0072	0.0102	0.1938
CL 15/19 4th	0	0.0072	0.0104	0.1938
CL16/20 G/2nd	0	0.0054	0.0073	0.1462
CL 16/20 3/4th	0	0.0054	0.0086	0.1281

8. CONCLUSION

An analytical model is presented to simulate the moment curvature behavior of reinforced concrete. Based on control of load increments, the algorithm proposed by Mander enables determination of moment – curvature-strain relationship with any geometry and material properties up to the maximum capacity of the section; however with a constant axial load or control of deformation increments, this model can be used to compute both the ascending and descending branches of the moment-curvature curve. The moment-curvature (or moment-rotation) relations play an important part in the study of limit analysis of two or three dimensional reinforced concrete frames.

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